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OPTIMIZATION OF MAINTENANCE COSTS OF A PIPELINE FOR A V-SHAPED HAZARD RATE OF MALFUNCTION INTENSITIES

OPTYMALIZACJA KOSZTÓW EKSPLOATACYJNYCH RUROCIĄGU DLA V-KSZTAŁTNEJ FUNKCJI INTENSYWNOŚCI USZKODZEŃ*

In this paper I focus on an evaluation of maintenance costs of a water distribution system (WDS), if a concept of a value of money in time is taken into account. Contrary to more classical approaches, instead of a constant yield, a strictly stochastic process (i.e., the one-factor Vasicek model) of an interest rate is assumed. Such an assumption presents uncertain, future behaviour of the yield in a more correct, realistic way. Moments of failures of connections in a WDS are generated using the Monte Carlo simulations via a new kind of a convex hazard rate function (HRF), which is proposed in this paper. Moreover, quality of a pipeline and a number of previous failures have direct influence on statistical properties of this introduced HRF. Apart from an analysis of the simulated output (like the maintenance costs), the Kiefer-Wolfowitz method is used for a better adjustment of one of parameters of a WDS – deterministic and unconditional replacement (i.e., planned replacement) time of each pipe. Algorithms, for both the simulations of the failure moments for the introduced HRF and the optimization step, are also provided. Additionally, some examples of a WDS for a crisp and a fuzzified settings are statistically analysed.

***Keywords**: water distribution system, maintenance costs, convex hazard rate function, Monte Carlo simulations, present value of money, Kiefer-Wolfowitz method, one-factor Vasicek model.*

W niniejszej publikacji skupiam się na obliczeniu kosztów eksploatacji wodociągu (water distribution system – WDS), jeśli pod uwagę zostanie wzięta wartość pieniądza w czasie. W przeciwieństwie do klasycznego podejścia, zamiast stałej wartości stopy procentowej, zakładam stochastyczny proces stopy procentowej (w postaci jednoczynnikowego modelu Vasicka). Założenie to przedstawia niepewne, przyszłe zachowanie stopy procentowej w bardziej dokładny i realistyczny sposób. Momenty awarii połączeń w WDS generowane są z wykorzystaniem metody Monte Carlo poprzez zastosowanie nowego typu funkcji intensywności uszkodzeń (hazard rate function – HRF), który zaproponowany został w niniejszej publikacji. Ponadto, jakość połączenia oraz ilość wcześniejszych uszkodzeń ma bezpośredni wpływ na statystyczne właściwości wprowadzonej HRF. Oprócz analizy wygenerowanych za pomocą symulacji wyników (takich jak koszty eksploatacji), użyta została metoda Kiefera-Wolfowitza w celu lepszego dopasowania jednego z parametrów WDS – deterministycznego i bezwarunkowego momentu wymiany każdego z połączeń (czyli wymiany planowanej). Zaprezentowane zostały również algorytmy zarówno dla symulowania momentów uszkodzeń przy użyciu zaproponowanej HRF, jak i dla kroku optymalizacyjnego. Ponadto, wykonana została analiza statystyczna kilku przykładów WDS dla dokładnych („crisp") i rozmytych („fuzzy") wartości parametrów.

***Słowa kluczowe:** system dystrybucji wody, koszty eksploatacji, wypukła funkcja intensywności uszkodzeń, wartość obecna pieniądza, metoda Kiefera-Wolfowitza, model jednoczynnikowy Vasicka.*

1. Introduction

From the customers' point of view, the main aim of a water distribution system (which is further abbreviated as WDS) is to deliver water, moreover – water of desirable quality and in necessary quantity. Therefore, different maintenance services have to be performed, e.g., broken or simply malfunctioned pipes or other parts of a WDS should be repaired or replaced. Because water is an indispensable good for humans, therefore also scientific literature devoted to reliability of water distribution systems is abundant. Firstly, let us mention reviews of various methods, approaches and literature, which can be found in, e.g., [15,28,29]. The papers themselves are very varied – some of them concern hydraulic and physical characteristics of parts of a WDS (see, e.g., [4,17]), other discuss rather a "macro-management" of a WDS rehabilitation problem (see, e.g., [12,22]) or only a "micro-management" scale (e.g., for a single building, see [1]). Even some monitoring systems for failures detection in a WDS are proposed (see, e.g., [23]).

Usually, if maintenance costs for a WDS are considered, planning for a relatively long-time horizon should be taken into account. Such a time interval covers 20, 50 or even 60 years (see, e.g., [12]). Of course, one unit of money, which is paid now, and the same unit in 50-60 years, are not equal. Therefore, an influence of a future / present value of the money onto a calculation of the maintenance costs should be taken into account. However, most of the authors apply only a constant interest rate, as a discount factor, to calculate a present value of future cash flows. Such an assumption is, of course, too strong and unrealistic in practical situations. Therefore, in this paper I adopt a more realistic and complex model – a variable interest rate, which is described by the widely known one-factor Vasicek model.

Moreover, some model for intensities of malfunctions of parts of a WDS has to be assumed. As it is proposed in literature, it can be based on selected physical aspects of a pipe and numerical equations (like the Hazen-Williams equation, see, e.g., [12]), it can be described with some type of Markov or semi-Markov process (see, e.g., [13, 16, 26]) or malfunctions are randomly generated using a hazard rate function (HRF). A

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

multilateral review of various HRFs can be found in [29]. In this paper I propose a new kind of a HRF, which can be easily adapted to real-life data and which is very efficient during the Monte Carlo simulations.

Very often in literature, some optimization procedure for minimization of the maintenance costs of a WDS is proposed. For example, the total costs of a renewal, risk and an unavailability of a WDS is given as one function of time and then minimised (see, e.g., [24]), different scenarios with and without replacement of pipes for the installation and repair costs with a damage and inconvenience cost multiplier are considered (see, e.g., [18]) etc. In the following, I also propose an optimization approach. During this procedure, the Kiefer-Wolfowitz algorithm is applied to find a minimum of the maintenance costs, if an unconditional replacement age of a pipe is our variable parameter.

Also some model of the previously mentioned maintenance costs has to be adopted. In literature, these costs are related to various sources and models, like a rehabilitation of a pipe and breakage repair costs (see, e.g., [12]), an extra energy, water losses and a loss of revenues (see, e.g., [11]) etc. In this paper the costs are modelled using their constant and variable part. This second element is related to time, which is necessary to conduct a repair or a replacement of a malfunctioned pipe. However, in some papers such a concept (i.e., time of a service) is completely neglected. Such a simplification is possible (see, e.g., [29] for a more detailed discussion), but usually the relevant period is modelled by some random variable (like the exponential distribution, see, e.g., [11]). In the following, I also assume, that a time, which is necessary for a repair or a replacement, is given by some random distribution.

This paper can be seen as a further development of some ideas, which were previously discussed in [25,26], but it is also a proposition of completely new ones. Hence, my current contribution is fourfold.

Firstly, I propose a new kind of a hazard rate function, which describes the intensities of malfunctions of pipes in a WDS. Many different HRFs were discussed in literature, however, each of them has some significant disadvantages (see, e.g., [29] for a comprehensive review). The HRF, which is introduced in this paper, has some appealing features. It is V-shaped, so it models two different states of a connection: a starting burn-in period (immediately after a repair or an installation of a pipe) and a later wear-out period (when an intensity of malfunctions for a connection is higher than during its starting phase). This HRF also depends on a number of previous repairs of the given pipeline, so an increasing deterioration of a material, which is caused by recurring stresses of repairs, can be taken into account. Moreover, a relevant algorithm for a random generation of intervals of times between malfunctions is numerically very efficient and straightforward. Therefore, the Monte Carlo simulations, which are then based on this HRF, can be directly applied to simulate behaviour of a whole WDS. Furthermore, the parameters of this HRF can be easily fuzzified, which enables us to introduce an additional source of an imprecision and an uncertainty, other than a strictly probabilistic one. These features should be highlighted, as important ones, when the introduced hazard rate function is compared to other HRFs.

Secondly, I adapt the Kiefer-Wolfowitz algorithm to find a minimum of the total maintenance costs. This method allows me to decide about an optimal value of a deterministic and unconditional replacement age, i.e., when it is better to replace a connection instead of its next, future repair. Because the mentioned algorithm directly utilizes stochastic nature of simulations, the Monte Carlo method can be directly used to calculate necessary estimators in this approach. In this paper, I focus on the unconditional replacement age as a variable, which is considered in the optimization problem, but the presented approach can be extended to other parameters, which are important for decision makers. Because an output, which is simulated in the analysis, behaves in a very varied and unpredictable way, I propose some practical alternation of the standard Kiefer-Wolfowitz algorithm. As it is pointed out in presented numerical examples, this optimization procedure can significantly lower the maintenance costs.

Thirdly, apart from a crisp case, I discuss a possible fuzzifaction of the parameters of the introduced HRF and of a model of the costs. As it is known (see, e.g., [3,6]), data can be imprecise and uncertain in real life situations. Moreover, sometimes it can not be appropriately modelled, if only a probabilistic approach is used. Therefore, I further develop an idea, which was discussed in [26], and some parameters of the assumed models are described by fuzzy numbers. It means, that they are not completely precise (i.e., "crisp") but they are, in some way, imprecise ("near to / about") and can be given as experts' opinions (even in a form of linguistic variables). For example, a cost of a repair is rather stated as "about 50 thousand (some unit of money)", than as a total and accurate value before this repair will take place. And a fuzzy setting is widely used in an analysis of possible financial decisions (see, e.g., [20,21]).

Fourthly, in contrary to [25,26], a more sophisticated model of time intervals of transitions between the states of a pipeline is proposed now. The intervals between the malfunctions are generated using the introduced HRF, and the times of repairs and replacements can be drawn from various probabilistic distributions. For a simplicity of the analysis, I focus on the exponential distribution, but other densities can be easily applied in the proposed simulation approach. Moreover, this model of the states is directly related to a model of costs of maintenance services. I distinguish two types of these costs (separately for a repair and a replacement) with two parts for each of them – a constant part (which is independent of length of time of a service) and a variable part (which depends on a random value of time of a repair or a replacement). Therefore, the considered model is closer to practical situations.

It should be pointed out, that the stochastic model of the interest rate (i.e., the one factor-Vasicek model, which is assumed in this paper) is directly embedded into the Monte Carlo simulations, as in [26]. To the best knowledge of the author, such an approach is still a new idea, which is not even considered in other papers. Whereas, there are significant differences in estimated values of costs and in other important results between models with a constant yield and with a variable discount factor, which is, of course, a more realistic assumption. Some of these discrepancies were already highlighted in [26]. Now I continue this analysis and show, that a simplified approach (i.e., with a constant yield) leads to different solutions and statistical results for the calculated maintenance costs. Therefore, future decisions can be also invalid, if such a too simplified model is assumed.

This paper is organized as follows. In Section 2, a new type of a hazard rate function, which describes times of the failures of a connection, is presented. A random generation procedure for a relevant density, which is based on this HRF, is also discussed. In Section 3, a model of possible states of an each connection with an additional parameter – deterministic and unconditional replacement age – is introduced. Section 4 is devoted to a description of maintenance costs, which are divided into constant and variable parts. Section 5 presents the Kiefer-Wolfowitz algorithm, which is then applied to optimize the discounted value of the maintenance costs in examples in Section 6. Apart from a numerical analysis of an example in a crisp case, a proposition of fuzzification of some parameters of the model is also examined using the Monte Carlo simulations. Results, which are obtained in these examples, are statistically summarized then. The paper is concluded in Section 7 with some final remarks.

2. Model of failure intensities

Let us suppose, that a WDS is modelled by a graph of connections G. In this graph, each connection (i.e. a pipeline which is a part of the whole WDS) is represented as an edge, and possible sources or outflows are denoted by nodes. In the following, I focus only on the

edges of the graph G, i.e. the connections of the considered WDS. Let us assume, that these connections behave in a statistically independent way, i.e. there is no "information flow" between the connections and time of a malfunction of one pipe does not influence on quality and possible malfunctions of other connections.

Firstly, I assume that times of the failures for an each connection are described by a hazard rate function (HRF or simply a hazard function) $\lambda(x|n_r)$, given by the formula:

$$\lambda(x|n_r)=\begin{cases}-a_0 x+a_0 x^*+y^*+\alpha_r n_r & if \quad x\in\left[0,x^*\right)\\ a_1 x+y^*-a_1 x^*+\alpha_r n_r & if \quad x\geq x^*\end{cases}, \qquad (1)$$

where $a_0>0, a_1>0, x^*>0$, $y^*>0, \alpha_r>0$ are parameters, which are related to the given type of a connection. Strictly speaking, such a HRF has a V-curve, linear shape (see Figure 1), for which:

- $-a_0$ is a directional component of a descending, linear part of the HRF (i.e., a left hand side of the function, for which $x\in\left[0,x^*\right)$),
- a_1 is a directional component of an ascending, linear part of the HRF (i.e., a right hand side of the function, for which $x\geq x^*$),
- $\left(x^*,y^*\right)$ is a point, where the HRF becomes an ascending linear function, instead of being a descending one,
- α_r is a parameter of deterioration of the connection related to a single, previous malfunction,
- n_r is a number of previous malfunctions of the connection, if there were repairs afterwards.

It is assumed that, when some connection is replaced with a completely new component, then $n_r=0$ is set for such a part. Hence, the parameter α_r reflects a level of fatigue, which is caused by previous malfunctions and repairs, without a replacement of such a connection. It directly increases a value of the hazard function $\lambda(x|n_r)$. The point $\left(x^*,y^*\right)$, and especially the value x^*, depends on time, when the HRF (1) changes its behaviour. In this point, instead of a burn-in period after some repair (or an installation of a new pipe), the connection reaches its wear-out period (see also, e.g., [5] for a more detailed descriptions of such states). It means, that for the first part of (1) an intensity of the malfunctions decreases, and for the second part this value increases with passing time, which approximates real-life situations in better way. Hence, the proposed function (1) can be used in straightforward manner to describe the intensity of malfunctions, taking into account two completely different quality states and progress of connection fatigue, which is also related to the number of previous repairs n_r. Therefore, this HRF can be better adjusted to real-life data.

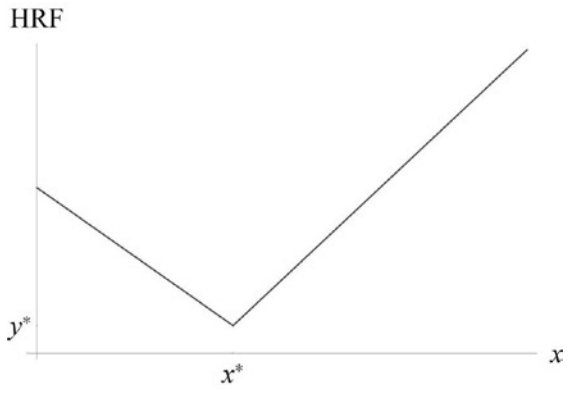


Fig. 1. Exemplary plot of the introduced HRF

From now on, for a simplicity of formulas, I use abbreviations:

$$b_0=a_0 x^*+y^*+\alpha_r n_r \ , \ b_1=y^*-a_1 x^*+\alpha_r n_r \ .$$

For a HRF, we have a general formula:

$$\lambda(x)=\frac{f(x)}{R(x)} \ ,$$

where $f(x)$ is a pdf (probability density function), $R(x)=1-F(x)$ and $F(x)$ is a cdf (cumulative density function), then in the case of (1), we get:

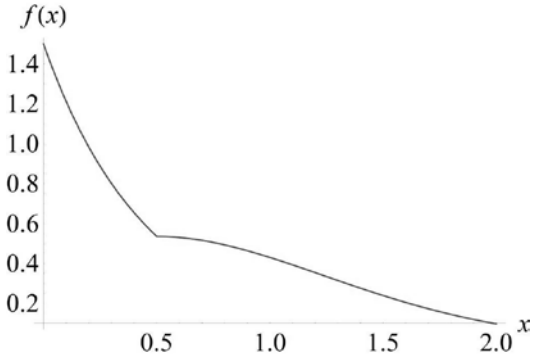
$$f(x)=\begin{cases}(-a_0 x+b_0)\exp\left(\frac{1}{2}a_0 x^2-b_0 x\right) & if \quad x\in\left[0,x^*\right)\\ (a_1 x+b_1)\exp\left(-\frac{1}{2}a_1 x^2-b_1 x-c_1\right) & if \quad x\geq x^*\end{cases}, \qquad (2)$$

where:

$$c_1=-\frac{1}{2}a_1\left(x^*\right)^2-b_1 x^*-\frac{1}{2}a_0\left(x^*\right)^2+b_0 x^* .$$

An exemplary plot of this density can be found in Figure 2. As it is seen, $f(x)$ is a continuous function with a visible point of a change of its behaviour (which is given by the parameters $x^*=0.5, y^*=1$ in this case).

Fig. 2. Exemplary plot of the introduced density



In Section 6, an analysis of a simulated output for times of malfunctions given by the density (2) is presented. Therefore, it is necessary to provide an efficient algorithm for a generation of random variables for such a pdf. It can be done using the composition method and the inversion method (see, e.g., [27] for an introduction and a review). In the composition method, a pdf $f(x)$ is decomposed as:

$$f(x)=\sum_{i=1} f_i(x)p_i \ ,$$

where, for an each $i = 1,2,\ldots$, $f_i(x) \geq 0$ is some density and $p_i \geq 0$ is a discrete probability. In the case of (2), we have:

$$p_1 = P\left(X \in \left[0, x^*\right)\right) = 1 - \exp\left(\frac{1}{2} a_0 \left(x^*\right)^2 - b_0\, x^*\right)$$
$$p_2 = P\left(X \geq x^*\right) = 1 - p_1, \tag{3}$$

and the pdfs $f_1(x)$ (for $x \in \left[0, x^*\right)$), $f_2(x)$ (for $x \geq x^*$) lead to relevant inversions of their cdfs, which are equal to:

$$F_1^{-1}(y) = \frac{b_0 - \sqrt{b_0^2 + 2a_0 \ln(1 - p_1 y)}}{a_0},$$
$$F_2^{-1}(y) = \frac{-b_1 + \sqrt{\left(a_1 x^* + b_1\right)^2 - 2a_1 \ln(1 - y)}}{a_1}. \tag{4}$$

Therefore, a simulation of random times of the failures is straightforward (see Algorithm 1). Moreover, because the inversion method is applied, the whole algorithm is numerically very efficient. None of the randomly generated points are rejected, as it is commonly seen in, e.g., the ROU (ratio-of-uniforms) method.

Algorithm 1 (Generation procedure for the HRF)
Input: A set of the parameters of the HRF (1).
Output: A random time of a failure *X*.

Calculate p_1, p_2 *, which are given by (3);*
Generate independent random values U, Y *from the uniform standard distribution* $U[0;1]$*;*
if $U \leq p_1$
$X = F_1^{-1}(Y)$ *(see (4));*
else
$X = F_2^{-1}(Y)$ *(see (4));*
end
return *X*

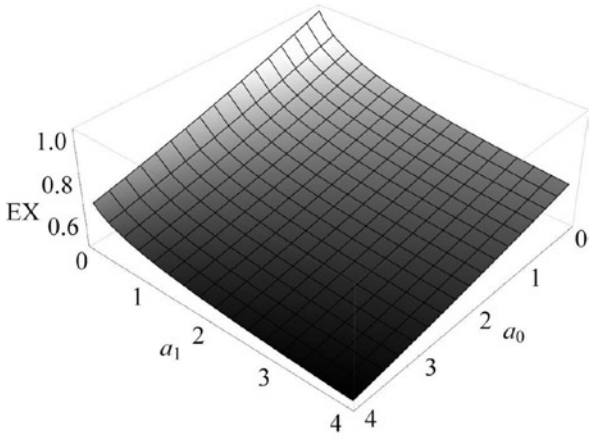


Fig. 3. Plot of the expected value (EX) for the introduced density as a function of a_0 *and* a_1 *.*

Because of a few parameters, which describe the formula (1), the introduced model of time of a failure can be applied in many various cases. Exemplary plots of the expected values for the relevant density (2) can be found in Figure 3, Figure 4 and Figure 5. As it is seen, the expected value of time of a failure is both a linear and non-linear function, which simplifies an adjustment to complex real-life data.

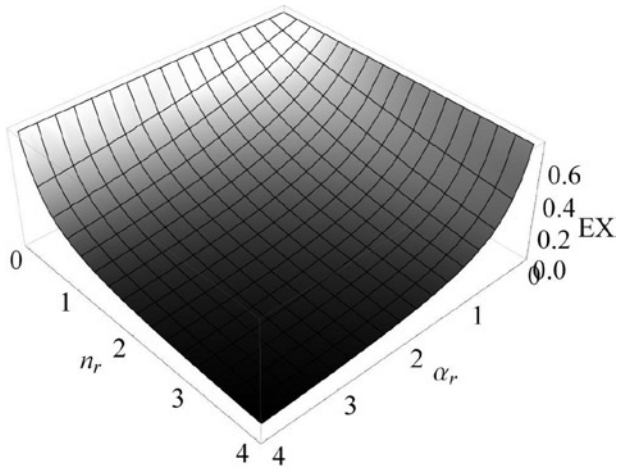


Fig. 4. Plot of the expected value (EX) for the introduced density as a function of α_r *and* n_r *.*

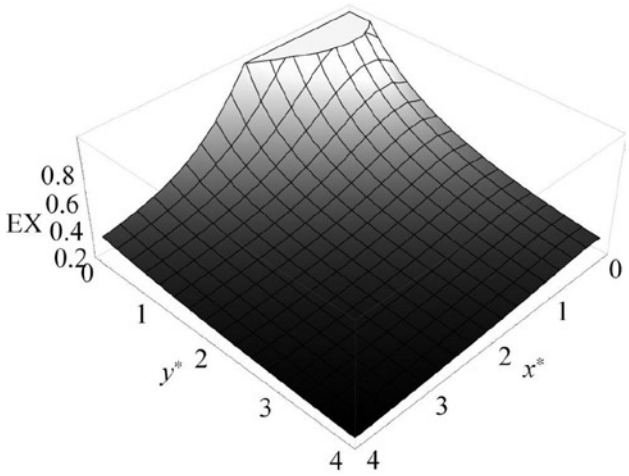


Fig. 5. Plot of the expected value (EX) for the introduced density as a function of x^* *and* y^* *.*

3. States of a connection

An each connection in time *t* can be in one of the following states: *working, under repair, under replacement*. It means, that immediately after some failure, a connection is repaired or replaced by a new one.

A random length of working time WT_i, after a repair or a replacement of the pipe and before a next malfunction, is given by (2). A length of repairing time RT_i (after a malfunction, when a connection is being repaired) can be modelled by various random distributions, e.g. the exponential distribution or the lognormal one. Of course, this distribution and its parameters should be fitted to real-life data, e.g. using statistical methods. The same applies for a length of replacement time PT_i (i.e., after a malfunction, when a connection is being replaced with a new one).

As in [26], I introduce a deterministic and unconditional replacement age P^*. This value is used to decide, if instead of one more repair, the connection in question should be rather replaced. It means that, when:

$$\sum_{i=1}^{j} WT_i + RT_i > P^*, \tag{5}$$

where $WT_1,\ldots,WT_j$ and $RT_1,\ldots,RT_j$ are working and repairing times after the last replacement of a connection, then this connection is replaced with a new one. Afterwards, $n_r = 0$ is set in (1), so such a replacement "restarts" a deterioration process.

4. Maintenance costs

As it was noted in Section 2, it is possible to directly simulate the periods of the working times WT_i of the considered connection for the pdf given by (2). Also, if numerically feasible distributions for the repairing times RT_i and the replacement times PT_i are selected (like, e.g., the lognormal distribution), then the Monte Carlo approach can be applied. Furthermore, the replacement condition (5) can be easily embedded in such a setup, without a necessity of conducting of complex theoretical probabilistic calculations.

Hence, the MC approach can be applied to generate the subsequent states of each connection *j*, and then, in a similar way, to simulate behaviour of the whole WDS. From now on, I assume, that these connections behave in a statistically independent way. However, if there is some kind of dependency, the MC procedure can be also used. Easily seen, apart from WT_i, RT_i and PT_i, exact times of the malfunctions can be found, when the necessary maintenance services (i.e., replacements or repairs) begin. In the following, these times are denoted by $t_1, t_2, \ldots$.

In this paper I focus only on the maintenance costs related to the replacements and the repairs. Of course, other types of costs (like costs of water losses, loss of revenues etc. – see, e.g., [5,11,12,18]) are commonly considered in the literature. Among others, I should also mention restoration and diagnostic costs. They are very important, especially for long time horizon of an analysis. Some of the mentioned costs can be easily taken into consideration using the MC approach. It seems, that this is also possible for the restoration and the diagnostic costs. However, due to nature of further assumptions in this paper, these costs can be rather related to the HRF itself, instead of time of a service (like a repair or a replacement). For example, after a restoration of a connection, a value of n_r can be lowered or values of a_0 and a_1 can be respectively changed for this connection. And an aim of such a change would be to increase length of a period to a next malfunction given by (1). But still the simulation approach is appropriate in this case. Of course, incorporation of other types of the costs (like restoration costs) can have some influence on the obtained results.

I assume, that the mentioned costs depend on a type of a service (i.e., if it is a replacement or a repair), length of such a service and a type of the considered connection. Therefore we have:

$$c^{(j)}(t_i) = c^{(j)}_{R,const} + c^{(j)}_{R,Var}(RT_i)$$

or

$$c^{(j)}(t_i) = c^{(j)}_{P,const} + c^{(j)}_{P,Var}(PT_i),$$

where $c^{(j)}(t_i)$ denotes a total sum of costs for the given j-th connection and time t_i, when a necessary service begins, $c^{(j)}_{R,const}$ is a constant value independent of length of a period for a repair (or for a replacement in the case of $c^{(j)}_{P,const}$), i.e. it is a fixed cost, and $c^{(j)}_{R,Var}(.)$ denotes a variable cost of a repair (or a replacement for $c^{(j)}_{P,Var}(.)$), i.e. some function of length of this service. If the MC approach is applied, then the variable costs can be modelled in various ways, e.g., an additional random distribution, which is related to RT_i or PT_i, can be used.

As it was mentioned, I assume that the value of money depends on time in the considered setting. Therefore, the concept of a present value (or a future value), which is widely known in financial mathematics, is applied (see, e.g., [7,27]). It is especially useful, if we are interested in a long time horizon T (like 20 or even 50 years) for which the estimated costs of the maintenance services should be calculated. And these costs, for different management decisions and possible scenarios, can be easily compared for the same, present time, i.e. $t = 0$. It leads to a straightforward way to select the best decision, taking into account a financial risk.

To calculate the present value of the total sum of the costs of repairs and replacements:

$$PV(c) = \sum_{i,j} PV\left(c^{(j)}(t_i)\right), \tag{6}$$

some model of an interest rate should be used, in order to find a discounting factor $PV(.)$ for each $c^{(j)}(t_i)$. In the following, the one-factor Vasicek model (see, e.g., [7]):

$$dr_t = a(b - r_t) + \sigma\, dW_t\ , \tag{7}$$

is used, where r_t is a value of the interest rate at time *t*, W_t is the standard Brownian motion, and a, b, σ are parameters of this model. Moreover, *b* characterizes a long term mean level (i.e. the trajectory of r_t is directed to this value during its long run), *a* reflects speed of reversion towards *b*, and σ is an instantaneous volatility (variability) of the trajectory introduced by the random component W_t.

In the MC setting, a relevant iterative scheme for a generation of increments Δr_t of the process (7) should be used (see, e.g., [7]). The values of r_t for the fixed moments $0 = t_0 < t_1 < \ldots < t_n$ are given by:

$$r_{t_{i+1}} = exp\left(-a(t_{i+1} - t_i)\right) r_{t_i} + b\left(1 - exp\left(-a(t_{i+1} - t_i)\right)\right) + \sigma \sqrt{\frac{1 - \exp\left(-2a(t_{i+1} - t_i)\right)}{2a}} Z_i,$$

where $Z_1, Z_2, \ldots, Z_n$ are *iid* samples from *N(0,1)*. Also a cumulative factor:

$$fv_{(t_i, t_{i+1})} = \int_{t_i}^{t_{i+1}} r_s ds\,,$$

which is necessary to evaluate the present value, can be easily approximated (see, e.g., [7,26,27] for a more detailed discussion).

As it is pointed out in [26], if a variable interest rate is assumed (like the one-factor Vasicek model in this paper), then obtained results

are different from an output for a model with a constant interest rate. I will also show these differences in examples in Section 6.

5. Optimization procedure

In numerical examples, which are presented in Section 6, I am interested in various statistical measures, which are important for an analysis of the maintenance costs. In practical situations, a decision maker is also confronted with different scenarios, concerning values of some parameters. Because of stochastic nature of the introduced process of the interest rate (7) and behaviour of the WDS itself, a relevant optimization procedure is necessary. There are various methods, which can be used to solve the mentioned min-max problem with a stochastic background (see, e.g., [9]). However, in the following, I apply the Kiefer-Wolfowitz (KW) algorithm (see, e.g., [2]) with some alternations, which are necessary for the considered setting.

In general, an iteration scheme of the KW algorithm is based on a formula:

$$X_{n+1} = X_n - a_n \left(\hat{f}\left(X_n + c_n\right) - \hat{f}\left(X_n - c_n\right) \right) / c_n \ , \qquad (8)$$

where X_1 is an initial value, a_n and c_n are two real-valued, deterministic tuning sequences, and $\hat{f}\left(X_n + c_n\right)$, $\hat{f}\left(X_n - c_n\right)$ are estimators (which are usually based on the MC approach) of a goal function $f\left(.\right)$ for $X_n + c_n$ and $X_n - c_n$. The aim of the sequence (8), which is produced by this algorithm, is to minimize the value of $f\left(x\right)$, taking into account the decision parameter *x*. Speed and quality of a convergence to this minimum depend on the tuning sequences a_n and c_n (see, e.g., [2] for a more detailed discussion of some necessary requirements for these sequences).

Usually, the estimators $\hat{f}\left(X_n + c_n\right)$, $\hat{f}\left(X_n - c_n\right)$ are based on only single Monte Carlo samples, which are drawn from the relevant functions $f\left(X_n + c_n\right), f\left(X_n - c_n\right)$. But in some cases, a more sophisticated approach is necessary. As in the considered setting, a function $f\left(x\right)$ can behave in a very varied and unpredictable way, because of its stochastic nature. Therefore, the mentioned estimators, which are calculated as standard Monte Carlo averages, should be based on larger samples. Additionally, it can be profitable to store estimated values of $f\left(.\right)$ for previous steps of the algorithm, not only for the last one.

In the following, the numerical experiments are focused on an optimization of the maintenance costs, if the unconditional replacement age P^* is a decision parameter in (8). Strictly speaking, my aim is to find:

$$\min_{P^*} EPV\left(c\right) , \qquad (9)$$

i.e. a minimum of the expected, present value of the total sum (6), if P^* is a decision parameter. From a practical point of view, the unconditional replacement age is very significant for a decision maker, especially if a long time horizon is taken into account. Of course, other characteristics of the WDS can be also treated as decision parameters, but the presented approach is applicable in these cases, too.

6. Example of numerical analysis

Now I apply the KW algorithm and the Monte Carlo simulations to find an optimum value of the unconditional replacement age P^*, which is a solution of the problem (9). In order to do this, I present a simplified example, but similar to a real-life case. In this analysis, the HRF, given by (1), is used to simulate times of the failures.

I will start from a general description of the parameters in Section 6.1. Then, in Section 6.2, I will present assumed numerical values of these parameters for a strictly crisp case. These values are used further, in Section 6.3, to find an optimal value of P^* in the considered optimization problem. Also some other statistical measures of the maintenance costs are estimated there. In Section 6.4, I will discuss possible problems with suboptimality of the obtained solution. A dependency between the assumed model of an interest rate and the optimal solution for P^* is also considered there. Then, in Section 6.5, I will recall basic definitions and notation concerning a fuzzy approach. This fuzzy approach will be used in two analyses afterwards: firstly, when some parameters of the introduced HRF are fuzzified (Section 6.6), and secondly, to decide, if the estimated output is more prone to impreciseness related to the constant or to the variable parts of the costs (Section 6.7). The results of all of the analyses will be summarized in Section 6.8. I will conclude this example with some remarks about a possibility of using the presented approach in practical application (Section 6.9).

6.1. General description of the parameters

Taking into account the previous considerations, the parameters of the whole model, which are used in simulations, can be divided into four groups:

1. parameters of the given type of the connection, which are related to the HRF given by (1), i.e. $a_0, a_1, x^*, y^*, \alpha_r, n_r$,
2. parameters, which depend on the type of the connection, its location etc., and they are related to the maintenance costs $c_{R,const}, c_{P,const}, c_{R,var}\left(.\right), c_{P,var}\left(.\right)$ or to the lengths of times of necessary services (i.e. repairs and replacements), like parameters of the random distributions for RT_i and PT_i ,
3. parameters of the interest rate model, which are related to the financial setup (7), i.e. r_0, a, b, σ ,
4. other parameters, like P^* and time range for the whole simulation *T*.

6.2. Applied parameters for the crisp case

Firstly, I focus on the strictly crisp case. In a further numerical analysis, to simplify my considerations, I model a WDS, which consists of 20 connections of one type of a pipe. Let us assume, that one year is time unit, and $x^* = 0.5, y^* = 1$. It means, that after half of a year, the HRF of the time to a failure changes its behaviour and after a burn-in period, a connection is in its wear-out period. In general, these values can be given by some expert. And such a source combines an insight knowledge with a classical, probabilistic approach.

I also assume, that $a_0 = a_1 = 1$. It means, that the linear parts in the function (1) lean at an angle of 45 degrees and, if there is no previous repairs, the expected value of time to a next malfunction is equal to 0.715 of a year. This value can be easily found using numerical software, like, e.g., *Mathematica*. As it is seen from (1), if there are some previous repairs, the whole HRF shifts upward by a multiplication of the parameter α_r and the number of the previous repairs n_r . Let us assume, that $\alpha_r = 0.2$. Then, after one repair, the time to a next malfunction is shortened to 0.635 (about 11%).

The next set of the parameters is related to the maintenance costs. In the following analysis, I apply one monetary unit assumption and set $c_{R,const}=1, c_{P,const}=3, c_{R,var}(t)=100t, c_{P,var}(t)=100t$. Then, the constant cost of a replacement is three times greater than the constant cost of a repair, and the variable costs are linear functions of time, which is necessary for these services. Also, the variable cost of a replacement is the same as the relevant cost of a repair, which is equal to about 0.274 per day (plus the constant cost, which is paid once). Additionally, time of the maintenance service should be also modelled in some way. In the following, I apply the exponential random variable to describe both the time of a repair (with a parameter λ_R for its density) and the time of a replacement (with a parameter λ_P, respectively). Other random distributions, like, e.g., the lognormal distribution, are also useful and have a significantly practical meaning in this area. In this case, I set $\lambda_R=365$ (so, the expected value of the time of a repair is equal to one day) and $\lambda_P=182.5$ (then, the expected value of the time of a replacement is equal to 2 days).

Of course, in practical applications the relevant parameters of the considered connections should be estimated from real data (or based on the experts' opinions).

The last group of the parameters, which is mentioned in Section 6.1, describes the interest rate model. For the one-factor Vasicek model, which is analysed in this paper, it is assumed that

$$a=0.1, b=0.05, r_0=0.04, \sigma=0.001$$

and a very long fifty years horizon of the financial analysis of the maintenance costs is considered (i.e., $T=50$). Therefore, comparing with [26], even longer time period is taken into account.

6.3. Results of the optimization procedure in the crisp case

Now, when all of the necessary parameters are set, the KW algorithm for finding the solution of the problem (9) can be started.

In the following analysis, I use $P_0^*=5$ as an initial value. As it was mentioned in Section 5, because of stochastic nature of the estimators $\hat{f}\left(P_n^*+c_n\right)$, $\hat{f}\left(P_n^*-c_n\right)$, the KW algorithm has to be slightly modified. Therefore, to calculate the values $\hat{f}\left(P_n^*+c_n\right)$, $\hat{f}\left(P_n^*-c_n\right)$, $m=100000$ simulations are conducted for each of them and the relevant Monte Carlo averages are found.

After 50 steps of the KW algorithm, an optimal value of P^*, which solves the problem (9) is achieved. In the considered case, it is equal to 3.58. Now, let us compare various statistical measures of the maintenance services for this optimal value $P^{**}=3.58$ and the starting point $P_0^*=5$. I examine an estimator of the discounted costs of services ($\widehat{PV}(c)$), an average number of repairs ($\overline{x}_R$) and unconditional replacements ($\overline{x}_P$), a minimum cost of a repair ($\min c_R$) and an unconditional replacement ($\min c_P$), an average cost of repairs ($\overline{c}_R$) and replacements ($\overline{c}_P$), a maximum cost of a repair ($\max c_R$) and a replacement ($\max c_P$), a standard deviation of costs of repairs ($sd(c_R)$) and replacements ($sd(c_P)$).

As it is seen from Table 1, ($\widehat{PV}(c)$) is reduced about 3.39%, if the optimal value of P^* is used and a value of $\overline{x}_R$ is reduced even more, about 14.21%. It means, that the overall discounted costs are now smaller and the repairs are more rare. In contrary, a value of $\overline{x}_P$ is greater for P^{**} about 44.44%. Therefore, in this case, the more often unconditional replacements lead to a decrease of the number of repairs. Statistics for the cost of a single repair or a replacement are very similar for the both values of P^*, so they do not affect the obtained conclusions.

Table 1. Comparison of statistical measures of the maintenance services for the optimal value and the starting point

Measure	$P^{**}=3.58$	$P_0^*=5$
$\widehat{PV}(c)$	1307.52	1353.36
$\overline{x}_R$	1929.56	2266.41
$\overline{x}_P$	260	180
$\min c_R$	1	1
$\max c_R$	6.39567	6.22472
$\overline{c}_R$	1.27371	1.27373
$sd(c_R)$	0.273725	0.273755
$\min c_P$	3	3
$\max c_P$	12.3977	12.4351
$\overline{c}_P$	3.54815	3.54807
$sd(c_P)$	0.547971	0.548069

6.4. The optimization procedure – additional remarks

Of course, if the KW algorithm is applied, it is possible, that instead of an optimal point, some suboptimal value is found. However, it is not a case in the considered example. In Figure 6, $\widehat{PV}(c)$ is plotted as a function of P^* and denoted by circles. As it is seen, this function has a clear U-shape. On the other hand, Figure 7 allows us to analyse behaviour of the averages $\overline{x}_R$ (a plot denoted by squares)

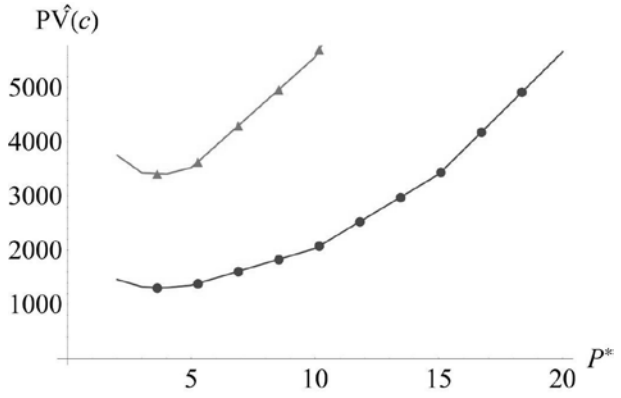


Fig. 6. Plot of $\widehat{PV}(c)$ for the one-factor Vasicek model (circles) and a nominal value (triangles) as a function of P^*

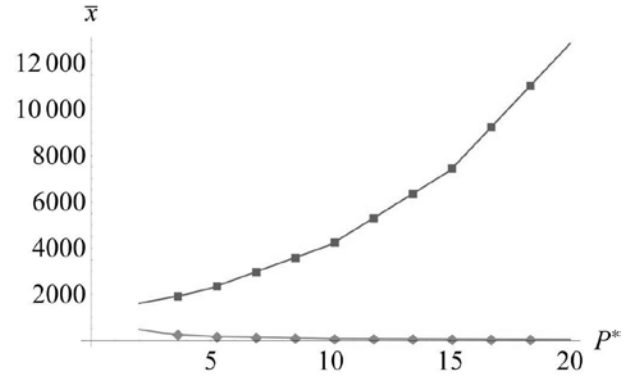


Fig. 7. Plot of $\bar{x}_R$ (squares) and $\bar{x}_P$ (diamonds) as functions of P^*

and $\bar{x}_P$ (a plot denoted by diamonds). The average number of repairs grows rapidly fast, if it is compared to a slow decrease of the average number of replacements. Then, clearly, it is fruitful for a decision maker to choose the calculated optimal value of P^*, instead of greater or lower one.

It was noticed in [26], that if a nominal value of the cash flow or a model with a constant yield is used, this leads to an incorrect estimation of the costs of the maintenance services. Because now I consider an optimization approach, a similar problem should be formulated – if other optimal value of P^* will be found, when the model, which describes the cash flows, is changed? I apply the KW approach also for the model with nominal values of the cash flows (i.e., a value of one unit of money is constantly the same, there is no discounting). Then, the optimal value of P^* is calculated as 3.34, which is about 6.7% lower than $P^{**} = 3.58$.

Also averages of the costs can be compared. In Figure 6, apart from $\widehat{PV}(c)$, for the one-factor Vasicek model, a similar average for a nominal value of the cash flow as a function of P^* is plotted and denoted by triangles. As it is easily seen, if a value of money is not taken into account, the nominal average cost of the maintenance services is overestimated and the optimal value of P^* is shifted to the left hand side.

6.5. Fuzzy approach – basic notation and definitions

As it is noticed in many papers (see, e.g, [6,8,10,26]), some sources of impreciseness can be easily modelled by a fuzzy approach, so a value of such an imprecise parameter can be based on expert's knowledge. This approach is especially very important, when data is sparse and various data analysis methods, like statistics, are not usable or even not possible. Then, taking into account opinions of the experts, the necessary parameters of the model can be described, e.g., as "the value of this parameter is about 5". Because these opinions have not completely precise forms (like real numbers), fuzzy numbers are an obvious model to describe such statements.

In [26], an important step in an application of a fuzzy setting for simulations of the maintenance costs was made. Now, I conduct similar analysis, but for the new model of time of the failures, which was proposed in Section 2.

I start form basic definitions and notation concerning the fuzzy approach, which will be used in the further part of the paper. Additional details can be found in, e.g., [14].

For a fuzzy subset $\tilde{A}$ of the set of real numbers R I denote by $\mu_{\tilde{A}}$ its membership function $\mu_{\tilde{A}} : R \to [0,1]$ and by $\tilde{A}[\alpha] = \{x : \mu_{\tilde{A}}(x) \geq \alpha\}$ the α-level set of $\tilde{A}$ for $\alpha \in (0,1]$. Then $\tilde{A}[0]$ is the closure of the set $\{x : \mu_{\tilde{A}}(x) > 0\}$.

A fuzzy number $\tilde{a}$ is a fuzzy subset of R for which $\mu_{\tilde{A}}$ is a normal, upper-semicontinuous, fuzzy convex function with a compact support. Then for each $\alpha \in [0,1]$, the α-level set $\tilde{a}[\alpha]$ is a closed interval of the form $\tilde{a}[\alpha] = [a_L[\alpha], a_U[\alpha]]$, where $a_L[\alpha], a_U[\alpha] \in R$ and $a_L[\alpha] \leq a_U[\alpha]$.

A left-right fuzzy number (which is further abbreviated as a LRFN) is a fuzzy number with the membership function of the form:

$$\mu_{\tilde{a}}(x) = \begin{cases} L\left(\dfrac{x-a}{b-a}\right), & x \in [a,b] \\ 1, & x \in [b,c] \\ R\left(\dfrac{d-x}{d-c}\right), & x \in [c,d] \\ 0, & \textit{otherwise} \end{cases},$$

where $L, R : [0,1] \to [0,1]$ are non-decreasing functions, such that $L(0) = R(0) = 0$ and $L(1) = R(1) = 1$. A triangular fuzzy number, denoted further by $[a,b,c]$, is a LRFN with the membership function of the form:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ \dfrac{x-c}{b-a}, & x \in [b,c] \\ 0, & \textit{otherwise} \end{cases}.$$

In my further investigation, behaviour of a function $f(x)$ plays a crucial role. In order to approximate a fuzzy output $\tilde{f}(\tilde{x})$ for some fuzzy parameter $\tilde{x}$, monotonicity of $f(x)$ should be checked. If $f(x)$ is an non-decreasing function, then for the given α, the left end point $f_L[\alpha]$ is approximated using the crisp value $x_L[\alpha]$. The same applies for $f_U[\alpha]$ and $x_U[\alpha]$. In contrary, if $f(x)$ is an non-increasing function, $x_U[\alpha], x_L[\alpha]$ are applied to evaluate $[f_L[\alpha], f_U[\alpha]]$ (see, e.g., [19,26]).

6.6. Numerical analysis for the fuzzified parameters x^*, y^*

After the strictly crisp case, its fuzzy counterpart can be discussed. First, I assume that x^*, y^* are modelled by triangular fuzzy numbers and all of other parameters are given as crisp values, which are the same as in Section 6.2. As it was indicated, both x^* and y^* can be based on experts' knowledge, therefore fuzzy numbers are obvious tool to model these parameters. In the following, I set $\tilde{x}^* = [0.25, 0.5, 0.75]$ and $\tilde{y}^* = [0.5, 1, 1.5]$. It means, that a horizontal coordinate of the point, where the introduced HRF (1), changes its

behaviour is "about 0.5", and its vertical coordinate is "about 1", with impreciseness equal to "plus / minus 50%". Using these fuzzy values $\tilde{x}^*, \tilde{y}^*$, the Monte Carlo estimator of the discounted cost of services $\widehat{PV}(c)$ for the previously estimated optimal point P^{**} can be found. As it is seen in Figure 8, the output is a LRFN, which is slightly right-skewed. Its support is equal to $[979.649, 1769.21]$, which is -25% and +35%, if this interval is compared to a core of $\widetilde{PV}(c)$ (and, at the same time, the crisp value of $\widehat{PV}(c)$, which was estimated in Section 6.2). Therefore, the parameters x^*, y^* have important impact on the estimated discounted costs.

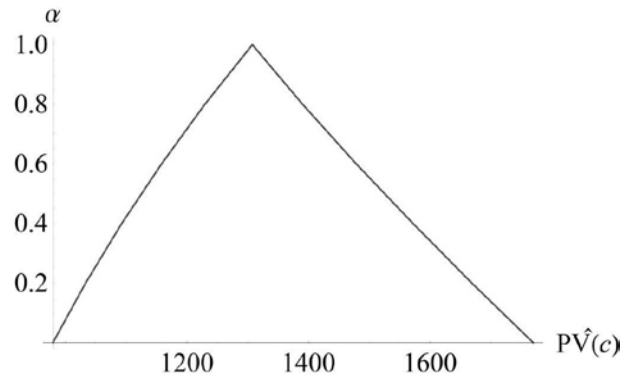


Fig. 8. Plot of $\widehat{PV}(c)$ as a fuzzy number for $\tilde{x}^* = [0.25, 0.5, 0.75]$ and $\tilde{y}^* = [0.5, 1, 1.5]$

Moreover, a fuzzification of the parameters x^*, y^* has an impact on the average number of the repairs. As it is seen in Figure 9, for $\tilde{x}^* = [0.25, 0.5, 0.75]$ and $\tilde{y}^* = [0.5, 1, 1.5]$, the relevant fuzzy average $\tilde{x}_R$ is also a LRFN, which is slightly right-skewed. Its support is equal to $[1271.46, 2859.13]$, which is -34% and +48%, if this interval is compared to the core of $\tilde{x}_R$. Therefore, a variability of $\tilde{x}_R$ is higher than for $\widetilde{PV}(c)$. It also means, that an optimal value of P^* should be found for each single analysed set of x^*, y^*. For example, 0-cut (i.e., α-cut of a fuzzy number, for which $\alpha = 0$) of $\tilde{P}^*$ is equal

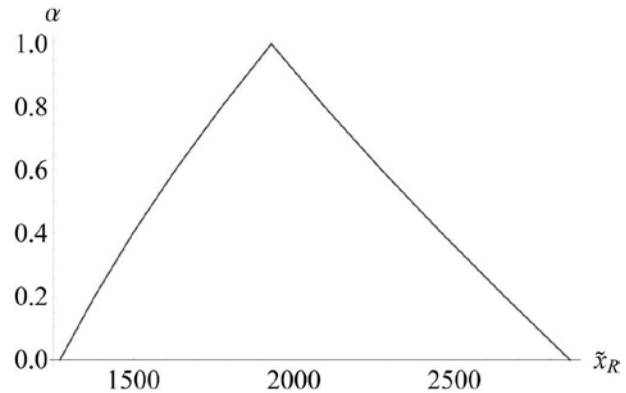


Fig. 9. Plot of $\tilde{x}_R$ as a fuzzy number for $\tilde{x}^* = [0.25, 0.5, 0.75]$ and $\tilde{y}^* = [0.5, 1, 1.5]$

Table 2. Comparison of statistical measures of the maintenance services for the left and the right end of $\tilde{P}^*[0]$.

Measure	$P_L[0] = 3.15$	$P_U[0] = 4.45$
$\widehat{PV}(c)$	1758.44	968.028
$\bar{x}_R$	2720.12	1389.72
$\bar{x}_P$	300	220
$\min c_R$	1	1
$\max c_R$	6.32746	6.01543
$\bar{c}_R$	1.27363	1.27373
$sd(c_R)$	0.273637	0.273719
$\min c_P$	3	3
$\max c_P$	12.2116	12.3969
$\bar{c}_P$	3.54795	3.54803
$sd(c_P)$	0.547883	0.548142

to $[3.15, 4.45]$ for the previously mentioned values of $\tilde{x}^*$ and $\tilde{y}^*$. Estimators of $PV(c)$ and statistical measures of the costs of repairs and replacements for the left and the right end of this 0-cut can be found in Table 2.

6.7. Numerical analysis for the fuzzified parameters of the costs

Apart from the parameters of the HRF, an influence of a fuzzification of other values can be analysed. For example, a decision maker can be interested in finding an answer, if the constant values of repairs and replacements or their variable counterparts are more prone to an impreciseness in an expert's opinion. Once again, the Monte Carlo simulations lead to a straightforward solution of this problem, which can be seen in Figure 10. In this case, I assume, that the relevant costs are "about plus / minus 10%", i.e. $\tilde{c}_{R,const} = [0.9, 1, 1.1]$, $\tilde{c}_{P,const} = [2.7, 3, 3.3]$ (the plot for the fuzzified constant values, which is labelled with squares) and $\tilde{c}_{R,var} = [90, 100, 110]$, $\tilde{c}_{P,var} = [90, 100, 110]$, (the plot with circles, for the fuzzified variable parts). As it is easily seen, the constant parts of the maintenance costs have more important impact on the estimated total costs. In both cases, the outputs are triangular fuzzy numbers. For the considered fuzzy constant costs, its support is given by +/− 1.98% (if it is compared to the "crisp" estimator of $PV(c)$) and in the case of the fuzzy variable costs, its support is more wider (+ / − 8%).

Of course, more than one source of impreciseness, which is modelled by a fuzzy approach, can be analysed. Using the Monte Carlo simulations, it is possible to observe interactions among many sources, e.g., the parameters of the HRF, the costs etc.

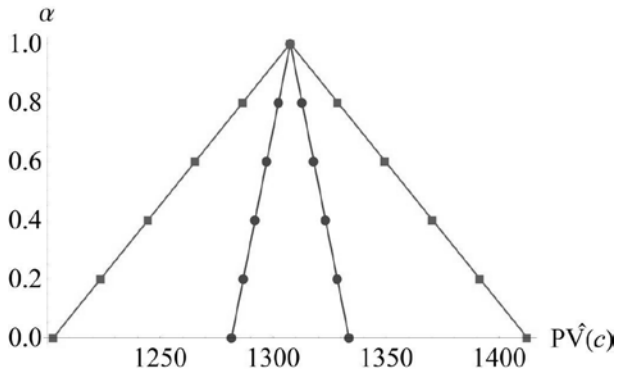


Fig. 10. Plot of $\widehat{PV}(c)$ *as a fuzzy number for* $\tilde{c}_{R,const} = [0.9,1,1.1]$, $\tilde{c}_{P,const} = [2.7,3,3.3]$ *(squares) and* $\tilde{c}_{R,var} = [90,100,110]$, $\tilde{c}_{P,var} = [90,100,110]$ *(circles)*

6.8. Summary of the results

Taking into account the previous analyses, some remarks about their results can be highlighted:

- In the considered case, the overall discounted costs of the services are smaller (about 3.39%) and the repairs are even more rare (about 14.21%) for the optimal solution. The costs of a single repair or a replacement are not affected by the optimization procedure.
- It is possible, that application of nominal values of the cash flow, instead of a more realistic model with a variable interest rate, leads to incorrect results for the optimization procedure just like other statistics of the costs.
- A fuzzification of various parameters of the introduced models is possible. For example, fuzzy values of $\tilde{x}^*$ and $\tilde{y}^*$ have a significant impact on the estimated discounted costs and the evaluated average number of repairs. Additionally, the optimization procedure should be applied for each single analysed set of these parameters.
- Using simulations and the fuzzy approach, many practical problems can be solved. In the considered fuzzy case, it turns out, that the constant parts of the maintenance costs have greater impact on the estimated total costs than their variable counterparts.

6.9. Towards a practical application

As it was mentioned, the previously presented example is a simplified one. However, some of its aspects can be directly carried to a real life application. For example, instead of only 20 connections of one type, a few hundreds or even a few thousands of connections together with many types can be considered. It can be achieved using the Monte Carlo simulations, because the HRF, which is introduced in this paper, is a numerically very efficient algorithm. The same applies for the optimization procedure, which is linearly dependent on the number of the samples (i.e. connections).

It seems, that a more complex problem is related to an estimation procedure for the applied parameters. If many connections are considered, then it can be quite laborious to estimate the necessary parameters of the HRF for each single type of a connection. In practical situations, it is also possible, that other kinds of the costs (see also the discussion in Section 4) and other effects (like a requirement to repair a whole group of connections in one time) should be taken into account. This directly leads to a more complicated model and longer time of the necessary simulations.

7. Conclusions

In this paper, a new kind of a hazard rate function for time between malfunctions of a pipeline is proposed. This HRF is a V-shape function, which also depends on number of previous repairs of the given connection. Moreover, times of malfunctions can be easily generated with the Monte Carlo simulations, if this HRF is applied. Then, a model of costs, which is related to a type of a performed service and its length, is also introduced.

These costs are dived into two parts, which facilitates an application of this approach in real-life situations. To calculate a present values of the maintenance costs, the one-factor Vasicek model is used. It is noticed during a more detailed analysis, that the obtained results strictly depend on the assumed type of a discount factor (i.e. if it is variable interest rate or a constant yield). During this numerical analysis, a behaviour of the whole WDS is simulated using the Monte Carlo approach. Afterwards, the costs of the maintenance services (i.e. repairs and replacements) are evaluated and statistically summarized. In this paper, a main aim of the conducted simulations is to minimize these maintenance costs. Therefore, an optimization procedure, which is based on the Kiefer-Wolfowitz algorithm, is applied. Apart from the strictly crisp setup, fuzzification of some parameters of the introduced models is also considered. Such an application of fuzzy numbers leads to a better incorporation of the experts' knowledge and more proper, closer to real-life situations, modelling of these uncertain parameters. Some relevant examples of the simulated output for both the crisp and the fuzzy settings are also provided.

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